(1) Which functions can be normalized?

$(a)\ f(x)=e^{ax}\quad a,x\in R$

$\int_{-\infty}^{+\infty}f^{*}(x)f(x)dx=\int_{-\infty}^{+\infty}e^{2ax}dx,\ \ does\ not\ exist$

$b)\ f(x)=e^{ax^{2}}\quad a,x\in R$

$\int_{-\infty}^{+\infty}f^{*}(x)f(x)dx=\int_{-\infty}^{+\infty}e^{2ax^{2}}dx=\begin{matrix}\sqrt{\pi/2|a|},\ a<0\\ not\ exist,\ a\geq 0\end{matrix}.$

$(c)\ f(x)=x^{n}e^{ax^{2}}\quad a,x\in R,\ n\in N_{0}$

$\int_{-\infty}^{+\infty}f^{*}(x)f(x)dx=\int_{-\infty}^{+\infty}x^{2n}e^{2ax^{2}}dx=\begin{matrix}\frac{1\cdot 3\cdots(2n-1)}{2^{n}}\bullet\sqrt{\pi/|2a|^{2n+1}},\ a<0\\ not\ exist\ a\geq 0\end{matrix}.$

$(d)\ f(x)=e^{iax}\quad a,x\in R$

$\int_{-\infty}^{+\infty}f^{*}(x)f(x)dx=\int_{-\infty}^{+\infty}1dx\quad does\ not\ exist$

$e)\ f(x)=\sin(x)\quad x\in R$

$\int_{-\infty}^{+\infty}f^{*}(x)f(x)dx=\int_{-\infty}^{+\infty}\sin^{2}xdx\quad does\ not\ exist$

$f)\ f(x)=\begin{matrix}\sin(x)\ x\in[0,2\pi]\\ 0\quad x\notin[0,2\pi]\end{matrix}$

$\int_{-\infty}^{+\infty}f^{*}(x)f(x)dx=\pi$

functions b), c) when a<0 and function f) are normalizable.

3) schrödinger equation

(a) The time independent wavefunction for free particlein one dimension is :

$$\varphi(x)=A\cos[2\pi/h(2mE)^{1/2}x]+B\sin[2\pi/h(2mE)^{1/2}x]$$

(b) The time dependent wavefunction:

$$\Phi(x.t)=\varphi(x)e^{-i2\pi Et/h}$$

(c) put the free particle in a one-dimensional box, by this we mean a particle subjected to a potential energy function that is infinite everywhere along the x axis except for a line segment of length l, where the potential energy is zero, then we can obtain:

$$\varphi(x)=(2/l)^{1/2}\sin(\frac{n\pi}{l}x),\ n=1,2,3\cdots$$