3. How to get harmonic oscillator model from morse potential around the region of equilibrium distance R_e?

Taylor expansion for morse potential about the point $R=R_e$:

$$V_m(R) = \sum_0^\infty \frac{V_m^{(n)}(R_e)}{n!}(R - R_e)^n$$

$$V_m^0(R_e) = 0,\ V_m'(R_e) = 0$$

$$V_m''(R_e) = k,\ V_m'''(R_e) = -3k\sqrt{\frac{k}{2D_e}}$$

$$V_m''''(R_e) = \frac{7k^2}{2D_e}, \ldots\ldots$$

$$V_m(R) = \tfrac{1}{2}k(R - R_e)^2 - \frac{k}{2}\sqrt{\frac{k}{2D_e}}(R - R_e)^3 + \frac{7k^2}{48D_e}(R - R_e)^4 + \ldots$$

$$\approx \tfrac{1}{2}k(R - R_e)^2$$